

## Lecture 4 «Heat transfer at a stationary mode. Heat transfer through a flat wall. Heat transfer through a cylindrical wall. Average temperature pressure. Determination of average temperatures of heat carriers. Thermal isolation»

**Aim:** Give output of the heat transfer equation through a flat and cylindrical wall. Characterize the average temperature difference and thermal insulation (isolation).

**Lecture summary:** Heat transfer is the heat exchange between two environments through the partition separating them. Heat transfer is a complex kind of heat exchange, in which two environments and a body participate. In addition, it operates simultaneously and together all the elementary phenomena of heat transfer (heat conductivity, convection, radiation).

Usually in the calculations one of the types of heat exchange is taken as the main one, and another type of heat exchange is secondary. For example, if the convective heat exchange is much higher than the radiant heat, it is taken as the main and the calculation formula for the general heat transfer is as follows:

$$q_o = (\alpha_c + \alpha_r)(t_{env} - t_s) = \alpha_o(t_{env} - t_s) \quad (1)$$

where  $\alpha_c$  – heat emission coefficient by convection;  $\alpha_r$  – coefficient of heat emission by radiation;  $\alpha_o$  – total heat emission coefficient;  $t_c$  – the temperature of the environment;  $t_s$  – the temperature of the wall surface.

The amount of heat transferred by heat transfer under steady-state conditions is determined by the basic heat transfer equation:

$$Q = KF\Delta t \quad (2)$$

where  $Q$  – the amount of transmitted heat, W;  $\Delta t = t_h - t_c$ , °C;  $t_h$  – the temperature of the hot heat carrier, °C;  $t_c$  – the temperature of the cold heat carrier, °C;  $F$  – heat exchange surface, m<sup>2</sup>;  $K$  – the heat transfer coefficient, the dimension of which is obtained from the basic equation:

$$[K] = \left[ \frac{Q}{K\Delta t} \right] = \left[ \frac{I}{s \cdot m^2 \cdot K} \right] = \left[ \frac{W}{m^2 \cdot K} \right] \quad (3)$$

The heat transfer coefficient is the amount of heat transferred per unit of surface per unit of time from one coolant to another with a temperature difference between them of one degree.

The heat transfer coefficient connects each other the coefficient of thermal conductivity and heat emission.

### Heat transfer through a flat wall

Let us consider the case when two environments of different temperatures are separated by a homogeneous flat wall, the width and height of which are sufficiently large in comparison with its thickness (Fig. 1). The coefficient of thermal conductivity of the wall –  $\lambda$  and its thickness –  $\delta$ . The temperature –  $t_{w_1}$  and  $t_{w_2}$ , and  $t_{w_1} > t_{w_2}$ . The temperature of the

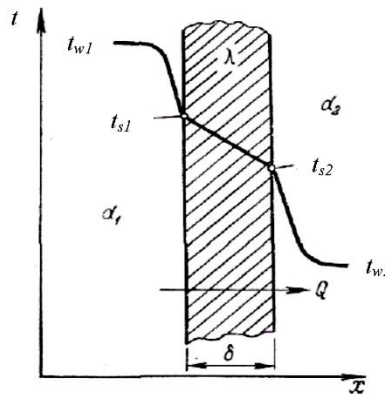
wall surfaces is unknown, we denote them as  $t_{s_1}$  and  $t_{s_2}$ . The total heat emission coefficient on the side of the hot heat carrier is  $\alpha_1$ , and on the cold one –  $\alpha_2$ .

By the condition of the problem, the temperature field is one-dimensional, the regime is stationary. In this case, all the heat transferred from the hot coolant to the surface of the wall passes through the wall and is released to the cool heat carrier, i.e. the indicated amounts of heat are equal to each other. Therefore, for a heat flow  $q$ , where  $q = \frac{Q}{F\tau}$ , we can write a system of their three equations:

$$\begin{aligned} q &= \alpha_1(t_{w_1} - t_{s_1}), \\ q &= \frac{\lambda}{\delta}(t_{s_1} - t_{s_2}), \\ q &= \alpha_2(t_{s_2} - t_{w_2}), \end{aligned} \quad (4)$$

Equations (4) contain partial temperature heads:

$$\begin{aligned} t_{w_1} - t_{s_1} &= q \frac{1}{\alpha_1}, \\ t_{s_1} - t_{s_2} &= q \frac{\delta}{\lambda}, \\ t_{s_2} - t_{w_2} &= q \frac{1}{\alpha_2}. \end{aligned} \quad (5)$$



**Fig. 1.** To the derivation of the heat transfer equation through a flat wall

After the left and right parts of equations (5) have been added, we obtain the expression for the total temperature head

$$t_{w_1} - t_{w_2} = q \left( \frac{1}{\alpha_1} + \frac{\delta}{\lambda} + \frac{1}{\alpha_2} \right) \quad (6)$$

from which the specific heat flow is determined:

$$q = \frac{t_{w_1} - t_{w_2}}{\frac{1}{\alpha_1} + \frac{\delta}{\lambda} + \frac{1}{\alpha_2}} \quad (7)$$

The term  $\frac{\delta}{\lambda}$  is the thermal resistance of the wall, and  $\frac{1}{\alpha_1}$  and  $\frac{1}{\alpha_2}$  are the thermal resistances of heat transfer from the hot heat carrier to the cold one.

According to formula (7), the heat flow is directly proportional to the temperature difference between the two heat carriers and inversely proportional to the sum of the thermal resistances.

Introducing the notation:

$$K = \frac{1}{\frac{1}{\alpha_1} + \frac{\delta}{\lambda} + \frac{1}{\alpha_2}} \quad (8)$$

in the expression (8), we obtain

$$q = K(t_{w_1} - t_{w_2}) \quad (9)$$

The quantity  $K$  is called the heat transfer coefficient. It establishes the relationship between the elementary types of heat exchange through the coefficients of heat emission and the coefficient of thermal conductivity.

The inverse value of the heat transfer coefficient is called the total thermal resistance of heat transfer:

$$\frac{1}{K} = \frac{1}{\alpha_1} + \frac{\delta}{\lambda} + \frac{1}{\alpha_2}, \quad (10)$$

where  $\left(\frac{\delta}{\lambda}\right)$  – the thermal resistance of the wall;  $\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2}\right)$  – are the thermal resistance of heat emission from the hot heat carrier to the cold one.

### Heat transfer through the cylindrical wall

The cylindrical wall separates hot and cold liquids with temperatures  $t_{w_1}$  and  $t_{w_2}$ . The temperatures of the wall surfaces are unknown, we denote them by  $t_{s_1}$  and  $t_{s_2}$ . The total heat emission coefficient from the hot liquid flowing inside the pipe is  $\alpha_1$ , and to the cold one –  $\alpha_2$ .

Under steady-state conditions, the amount of heat given off by the hot one and perceived by cold liquids is the same, and consequently, one can write:

$$\begin{aligned} q_l &= \alpha_1 \pi d_1 (t_{w_1} - t_{s_1}), \\ q_l &= \frac{2\pi\lambda}{\ln \frac{d_2}{d_1}} (t_{s_1} - t_{s_2}), \\ q_l &= \alpha_2 \pi d_2 (t_{s_2} - t_{w_2}). \end{aligned} \quad (11)$$

Solving these equations for the temperature difference, we obtain:

$$\begin{aligned} t_{w_1} - t_{s_1} &= \frac{q_l}{\alpha_1 \pi d_1}, \\ t_{s_1} - t_{s_2} &= \frac{q_l \ln \frac{d_2}{d_1}}{2\pi\lambda}, \\ t_{s_2} - t_{w_2} &= \frac{q_l}{\alpha_2 \pi d_2}. \end{aligned} \quad (12)$$

Adding the equations (12), we obtain the total temperature head

$$t_{w_1} - t_{w_2} = \frac{q_l}{\pi} \left( \frac{1}{\alpha_1 d_1} + \frac{1}{2\lambda} \ln \frac{d_2}{d_1} + \frac{1}{\alpha_2 d_2} \right) \quad (13)$$

Whence the value of the heat flow:

$$q_l = \frac{\pi(t_{w_1} - t_{w_2})}{\frac{1}{\alpha_1 d_1} + \frac{1}{2\lambda} \ln \frac{d_2}{d_1} + \frac{1}{\alpha_2 d_2}} \quad (14)$$

We introduce the following notation

$$K_l = \frac{1}{\frac{1}{\alpha_1 d_1} + \frac{1}{2\lambda} \ln \frac{d_2}{d_1} + \frac{1}{\alpha_2 d_2}} \quad (15)$$

After substituting this equality into (14), we finally obtain:

$$q_l = K_l \pi (t_{w_1} - t_{w_2}) \quad (16)$$

In contrast to  $K$ , the value  $K_l$  is a linear coefficient of heat transfer, per unit length of the tube, and not to the unit of its surface. Accordingly,  $K_l$  is expressed in  $W/(m^2 \cdot K)$ .

### Average temperature head. Determination of average temperatures of heat carriers

The processes of heat transfer at constant temperatures are relatively common. Such processes occur, for example, if steam condenses on one side of the wall, and on the other – boils up the liquid. Most often heat transfer in industrial equipment takes place at variable temperatures of heat carriers.

The temperatures of the heat carriers usually vary along the surface that separates their walls.

Heat transfer at variable temperatures depends on the mutual direction of motion of the heat carriers. In continuous processes of heat exchange, the following variants of the direction of motion of liquids relative to each other along the wall separating them are possible:

- 1) a parallel current, or direct flow, in which heat carriers move in the same direction;
- 2) a counter-current, in which heat carriers move in opposite directions;
- 3) a cross-current, in which heat carriers move mutually perpendicular to each other;
- 4) a mixed current, in which one of the heat carriers moves in one direction, and the other - both parallel (or direct) and counter-current to the first one.

The driving force of heat transfer processes at variable temperatures varies depending on the type of the mutual direction of motion of the heat carriers. Therefore, in the heat transfer equation, the average value of the temperature head

$$Q = KF \Delta t_m \quad (17)$$

The value of the average temperature head is determined by:

$$\Delta t_m = \frac{\Delta t_{in} - \Delta t_f}{\ln \frac{\Delta t_{in}}{\Delta t_f}} \quad (18)$$

Equation (18) remains valid also for determining the average logarithmic temperature head when the fluid moves counter-current.

If the temperature of the working fluids along the surface changes insignificantly, i.e. the condition is satisfied

$$\frac{\Delta t_{in}}{\Delta t_f} < 2,$$

then the average temperature head can be calculated as the arithmetic mean of the extreme heads

$$\Delta t_{av} = \frac{\Delta t_{in} - \Delta t_f}{2} \quad (19)$$

For mixed current and crosscurrent

$$\Delta t_m = \varepsilon_{\Delta t} \Delta t_{dir}, \quad (20)$$

where  $\varepsilon_{\Delta t}$  – the correction factor to the average temperature difference  $\Delta t_{cc}$ , computed for the counter-current.

### Thermal insulation

To reduce heat transfer, it is necessary to increase the thermal resistance. This is achieved by applying a layer of thermal insulation to the wall.

Thermal insulation is called any auxiliary coating, which helps to reduce the loss of heat to the environment. The choice and calculation of insulation is made taking into account considerations of an economic nature and the requirements of technology and sanitation.

The thickness of insulation for flat walls is determined directly from formula (7), and for pipelines from formula (14) through  $d_2$ , or the ratio  $\frac{d_2}{d_1}$ , where  $d_1$  – the diameter of the bare and  $d_2$  – insulated pipelines.

For pipelines, the determination of the insulation thickness is complicated by the fact that  $d_2$  enters the calculation equation not only in the form  $\ln \frac{d_2}{d_1}$ , but also in the form of the term  $\frac{1}{\alpha_2 d_2}$ . Thermal losses of insulated pipelines are reduced in proportion to the increase in insulation thickness. This circumstance is explained by the fact that as the thickness is increased, the thermal resistance of the insulation layer increases:

$$R_{in} = \frac{1}{2\lambda_{in}} \ln \frac{d_2}{d_1}, \quad (21)$$

and the thermal resistance of heat emission to the environment is reduced:

$$R_{\alpha} = \frac{1}{\alpha_2 d_2} \quad (22)$$

To avoid a large thickness in the insulation of pipelines, materials with a low coefficient of heat conductivity are used. The maximum thermal losses are observed at a certain value of the diameter, which is called the critical diameter of the insulation:

$$d_{2cr} = \frac{2\lambda}{\alpha_2}, \quad (23)$$

where  $d_{2cr}$  – critical insulation diameter at which there will be a maximum loss of heat, *mm*;  
 $\lambda$  – thermal conductivity of insulation;  $\alpha_2$  – coefficient of heat emission from the surface to the environment.

### Questions to control:

1. Give the basic heat transfer equation.
2. What is the physical meaning of the heat transfer coefficient?
3. Output the heat transfer equation through a flat wall.
4. What is the thermal resistance of heat transfer from a hot heat carrier to a cold one?
5. Output the heat transfer equation through the cylindrical wall.
6. List possible options for the direction of motion of liquids relative to each other along the wall separating them in continuous heat exchange processes.
7. Explain the physical meaning of the average temperature difference in the formula (17) in the calculation of heat exchangers.
8. What is called thermal insulation?
9. By what equation can the thermal resistance of the insulation layer be calculated?
10. What is the value called the critical diameter of insulation?

### Literature

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